Journal of Engineering Physics and Thermophysics, Vol. 72, No. 1, 1999

ENTROPY ANALYSIS OF FILTRATION FLOWS

I. M. Ametov

UDC 622.276

The author discusses the possibilities of studying filtration processes on the basis of an analysis of the entropy growth rate in the system in accordance with Prigogine's principle of minimum entropy production. Results of a study of the process of displacement of a liquid in a porous medium, the effect of surfactant additions on the process of displacement and isothermal flows, and the effect of natural magnetic fields on the characteristics of filtration flows are given.

Introduction. Analysis of the equation of entropy balance [1, 2] provides important information in studies of liquid and gas flows in porous media. It has special importance in considering multiphase flows with phase conversions, surface effects, etc. [2]. Use of the formalism of nonequilibrium thermodynamics has substantially expanded the types of filtration processes considered, in particular, it has made it possible to study flows accompanied by nonequilibrium phase conversions and relaxation phenomena. The main disadvantage of the approach to investigation of filtration flows is that local characteristics are mainly used. At the same time, consideration of flows in porous media at the global level, using integral characteristics, provides qualitatively new results. Here, additional possibilities for controlling filtration processes appear, which is of direct practical importance.

Prigogine formulated the principle of minimum entropy production under steady-state boundary conditions [3, 4]. According to this principle, the time evolution of a system occurs in such a way that as a whole the total entropy production tends to decrease and reaches a minimum in the steady state of the dissipative system. In what follows, some basic types of filtration flows are analyzed on the basis of this principle. In order to obtain qualitative results, all considerations are conducted within the simplest schemes, which, however, illustrate general properties.

1. Basic Equations. In what follows, in the derivation of the basic equations use is made of phenomenological relations of irreversible thermodynamics. The rate of local entropy production is equal to [3]

$$T\frac{dS}{dt}=\sum_{i=1}^{n}X_iI_i\,. \tag{1}$$

In considering filtration flows in an isotropic bed, assuming the validity of the Darcy law, it is possible to set [5, 6]

$$X=-\nabla P\,,\quad I=\overline{U}=-\frac{K}{\mu}\nabla P\,. \tag{2}$$

Isothermal flows will be considered. From Eqs. (1) and (2) we find

$$T\frac{dS}{dt}=\int_V\frac{K}{\mu}\,|\nabla P|^2\,dv\,. \tag{3}$$

Consider the process of displacement of one liquid (oil) by another (water, an aqueous solution) in a porous medium within the plug model. The filtration region G and the outer boundary Γ are divided into two

---

Deceased.

All-Russia Oil and Gas Research Institute, Moscow, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 71, No. 1, pp. 26-31, January-February, 1999. Original article submitted June 24, 1997.

parts: $G = G_1 + G_2$; $\Gamma = \Gamma_1 + \Gamma_2$, where the subscripts 1 and 2 refer to the displacing and displaced liquids, respectively. The interface between the liquids (the displacement front) is denoted by Γ_0. The volumes of the regions G_1 and G_2 are equal to V_1 and V_2, respectively.

Proceeding from relation (3), for the entropy production rate in the region G we have

$$T\frac{dS}{dt} = T\frac{dS_1}{dt} + T\frac{dS_2}{dt} = \int_{V_1} \frac{K}{\mu} |\nabla P|^2 dv + \int_{V_2} \frac{K}{\mu} |\nabla P|^2 dv \geq 0 .$$

Transforming the volume integrals into surface integrals in the above relation and taking into account that the integral along Γ_0 vanishes due to the continuity of pressure and filtration velocity, we obtain

$$T\frac{dS}{dt} = -\int_{G_1} \frac{K}{\mu_1} P\Delta P dg - \int_{G_2} \frac{K}{\mu_2} P\Delta P dg + \int_{\Gamma_1} \frac{K}{\mu_1} P \frac{\partial P}{\partial n} dl + \int_{\Gamma_2} \frac{K}{\mu_2} P \frac{\partial P}{\partial n} dl . \tag{4}$$

In view of the fact that the pressure distribution obeys the piezoconductivity equation and the integrals along Γ_1 and Γ_2 are equal to the injection and suction rates, instead of (4) we write

$$T\frac{dS}{dt} = -\int_{G_1} m\beta_1 P \frac{\partial P}{\partial t} dg - \int_{G_2} m\beta_2 P \frac{\partial P}{\partial t} dg + p_1 q_1 - p_2 q_2 . \tag{5}$$

In the derivation of relation (5) it was assumed that the pressures along the boundaries Γ_1 and Γ_2 are constant and equal to p_1 and p_2, respectively. Consideration of a more general situation in which, for example, the pressures at the contours of the wells are constant but different from each other does not introduce complications. In this case for an isolated bed, instead of (5) we have

$$T\frac{dS}{dt} = -\int_{G_1} m\beta_1 P \frac{\partial P}{\partial t} dg - \int_{G_2} m\beta_2 P \frac{\partial P}{\partial t} dg + \sum_{i=1}^{n} p_{1i} q_{1i} - \sum_{i=1}^{m} p_{2i} q_{2i} ,$$

where p_{1i}, p_{2i}, q_{1i}, q_{2i} are the pressures and discharges at the delivery and production contours (wells); n, m are the numbers of delivery and production contours (wells).

Instead of a linear filtration law, a more general case can be considered in which filtration of liquids 1 and 2 is described by nonlinear laws of the form

$$\overline{U}_1 = -\Phi_1 (|\nabla P|) \nabla P , \quad \overline{U}_2 = -\Phi_2 (|\nabla P|) \nabla P . \tag{6}$$

In this case only relations (2) are changed and instead of (4) we have

$$\frac{dS}{dt} = -\int_{G_1} P \operatorname{div} (\Phi_1 (|\nabla P|) \nabla P) dg - \int_{G_2} P \operatorname{div} (\Phi_2 (|\nabla P|) \nabla P) dg +$$

$$+ \int_{\Gamma_1} P\Phi_1 (|\nabla P|) \frac{\partial P}{\partial n} dl + \int_{\Gamma_2} P\Phi_2 (|\nabla P|) \frac{\partial P}{\partial n} dl . \tag{4a}$$

Relation (5) remains unchanged. Thus, the expression for the rate of change of entropy in the system takes the form of (5), irrespective of the form of the filtration law.

Similar relations are obtained if the model of plug displacement is rejected and two-phase filtration is considered. Repeating the preceding discourse, we can easily calculate the entropy growth rate for the present case:

$$T\frac{dS}{dt} = \int_V K \left[\frac{K_1(\sigma)}{\mu_1} + \frac{K_2(\sigma)}{\mu_2}\right] |\nabla P|^2 dv \geq 0 . \tag{7}$$

The volume integral is transformed into a surface integral:

$$T\frac{dS}{dt} = -\int_V P \operatorname{div}\left[K\left(\frac{K_1(\sigma)}{\mu_1} + \frac{K_2(\sigma)}{\mu_2}\right)\nabla P\right] dg +$$

$$+\int_\Gamma KP\left[\frac{K_1(\sigma)}{\mu_1} + \frac{K_2(\sigma)}{\mu_2}\right]\frac{\partial P}{\partial n}\,dl. \qquad (8)$$

In considering two-phase filtration, the compressibility of the liquids is usually neglected in comparison with the change in the saturation, i.e., the liquids are assumed incompressible [6]. In this case, due to the continuity equation, the volume integral in (8) is zero and we have

$$T\frac{dS}{dt} = \int_\Gamma KP\left[\frac{K_1(\sigma)}{\mu_1} + \frac{K_2(\sigma)}{\mu_2}\right]\frac{\partial P}{\partial n}\,dl. \qquad (9)$$

If we take into consideration that the filtration is unsteady, we should proceed from Eq. (5). Using average values, instead of (5) we write

$$T\frac{dS}{dt} = -\frac{\beta_1 V_{10}}{2}\frac{dP_1^2}{dt} - \frac{\beta_2 V_{20}}{2}\frac{dP_2^2}{dt} + q_1 p_1 - q_2 p_2 .$$

It follows from the above relation that in the case of increasing bed pressure, the entropy growth rate decreases, and vice versa. Thus, changes in the elastic energy of the oil pool are directly related to changes in the entropy growth rate. It follows from this reasoning that an increase in the pressure, including a local increase, for example, by stopping some of the production wells, causes a change in the filtration flows, redistribution of saturation, and a subsequent increase in the discharge of the wells. This fact is well known in practice and is a basis of hydrodynamic methods of increasing oil recovery [7].

2. Stability of Displacement in a Porous Medium. It is known that displacement of liquids in a porous medium is an unstable process, and within the scheme considered the condition of instability has the form $\mu = \mu_2/\mu_1 > 1$ [6]. In the case of unstable displacement, "finger formation" takes place and the process is fractal.

We find the relation between the entropy growth rate and the condition of stability of displacement. The process is assumed to be quasisteady. In this case $\partial P/\partial t = 0$ should be assumed in (5). Moreover, due to the condition $\Delta P = 0$ and accordingly $\int_G (K/\mu)\Delta P dg = \int_\Gamma (K/\mu)(\partial P/\partial n)dl = 0$ we have $q_1 = q_2 = q$. In sum, Eq. (5) is transformed to the form

$$T\frac{dS}{dt} = q\,(p_1 - p_2)\,. \qquad (10)$$

First, we consider the case of one-dimensional plug displacement. Following [8], it can easily be obtained that

$$\frac{dS}{dt} = aq^2\,[(1-\bar{l})\mu + \bar{l}]\,, \qquad (11)$$

where $a = \mu_1 L/KT$, $\bar{l}(t) = l(t)/L$, $\mu = \mu_1/\mu_2$.

From (11) we obtain

$$\frac{d^2S}{dt^2} = aq^2\,(1-\mu)\frac{dl}{dt}.$$

Since $dl/dt \geq 0$, the sign of d^2S/dt^2 is determined by the value of μ: at $\mu > 1$, the function $S(t)$ is convex and $d^2S/dt^2 \leq 0$; at $\mu < 1$, the function $S(t)$ is concave and $d^2S/dt^2 \geq 0$. Hence, it follows that when the displacement front loses stability and fractal structures are formed, the entropy growth rate increases. With the model considered the conditions $d^2S/dt^2 \le 0$, $d^2S/dt^2 \ge 0$ coincide with the conditions $\mu > 1$, $\mu < 1$. If the quantity $dS_0/dt = aq^2$, equal to the entropy growth rate in the region of water filtration with the same flow rate q, is introduced, for the relative entropy growth rate we have

$$\frac{dS}{dt}\Big/\frac{dS_0}{dt} = (1-\bar{l})\,\mu + \bar{l}\,. \tag{12}$$

The relations presented can be extended qualitatively to the case of non-one-dimensional filtration. The average pressure in the bed is taken to be $\bar{p}$. Equation (10) is rewritten in the form

$$T\frac{dS}{dt} = q\,(p_1 - \bar{p} + \bar{p} - p_2) = q\left(\frac{q\mu_1 l_1}{K} + \frac{q\mu_2 l_2}{K}\right) = \frac{\mu_1}{K}\,q^2\,(l_1 + \mu l_2)\,.$$

Here $l_1(t)$, $l_2(t)$ are some characteristic dimensions of the water and oil regions, $dl_1/dt \ge 0$, $dl_2/dt \le 0$. Just as in the one-dimensional case, assuming $T(dS_0/dt) = (\mu_1/K)q^2 l_0$, where l_0 is the characteristic dimension of the filtration region, we obtain

$$\overline{\frac{dS}{dt}} = \frac{dS}{dt}\Big/\frac{dS_0}{dt} = \bar{l}_1 + \mu\bar{l}_2\,, \quad \bar{l}_{1,2} = \frac{l_{1,2}}{l_0}\,. \tag{13}$$

The derivative of expression (13) is $d^2\bar{S}/dt^2 = d\bar{l}_1/dt + \mu(d\bar{l}_2/dt)$. Since $dl_1/dt \ge 0$, $dl_2/dt \le 0$, the sign of d^2S/dt^2 changes from minus to plus as the parameter μ increases. It follows from relations (12) and (13) that the entropy growth rate is a linear function of the parameter μ. Hence, it can be concluded that the "finger formation" that occurs in the displacement results in an increase in the entropy growth rate. It is evident that this is accompanied by a change in the structure of the filtration flow that is expressed in more frequent alternation of water and oil regions. Similar reasoning is also valid for a two-phase flow. This follows from the fact that relation (9) is transformed to the form of (11) $T(dS/dt) = Q(p_1 - p_2)$, where Q is the rate of water injection; $Q = Q_1 + Q_2$, $Q_{1,2}$ are the rates of water and oil intake, respectively.

3. Effect of Active Additions on the Process of Displacement. To improve the process of displacement in a porous medium, surfactants are added to the displacing liquid (water) to decrease the capillary pressure at the interface. When the capillary jump in the pressure between the phases is taken into consideration, an additional term appears in Eqs. (5) and (11) in Eqs. (5) and (11) that is caused by the inequality of the pressures along the common part of the boundary Γ_0 and is equal to q_0P_c, where q_0 is the liquid flow rate through the boundary Γ_0. In the quasisteady approximation considered, $q = q_0$. Thus, instead of (11) we have

$$\frac{dS}{dt} = aq^2\,[(1-\bar{l})\,\mu + \bar{l}\,] - \frac{1}{T}\,qP_c\,. \tag{14}$$

The minus sign in front of the second term in (14) corresponds to a hydrophilic porous medium. For a hydrophobic medium the sign is plus.

Several general conclusions follow from (14). When surfactants are added to injected water, P_c decreases, which results in an increase in dS/dt. However, here the viscosity of the displacing agent increases, which results in a decrease in the parameter μ, in particular, due to formation of an emulsion. As a result, dS/dt decreases. Hence, it follows that optimum concentrations of surfactants and conditions of displacement are possible as regards minimization of the entropy growth rate. Indeed, if the concentration of the reagent is denoted by c, we have from (14)

$$\frac{d}{dc}\left(\frac{dS}{dt}\right) = aq^2\,(1-\bar{l})\,\frac{d\mu}{dc} - \frac{q}{T}\,\frac{dP_c}{dc}\,. \tag{15}$$

TABLE 1. Results of Determination of the Permeability of Samples from the Koshilsk and Mamontovo Oil Fields

Sample No.	Direction of filtration	Gas permeability, 10^{-12} cm^2, in the direction		Water permeability, 10^{-12} cm^2, in the direction		Average increase in the water permeability	Oil field
		direct	reverse	direct	reverse		
27	West-east	2.77	2.79	0.46	0.46	1.14	Koshilsk
27	North-south	2.86	2.83	0.41	0.43		
30	West-east	2.82	2.86	0.49	0.48	1.55	
30	North-south	3.21	3.08	0.35	0.38		
1	West-east	337.7	342.6	59.6	60.9	2.04	Mamontovo
1	North-south	281.0	247.4	23.5	24.9		

The relations $d^2\mu/dc^2 \le 0$, $d^2P_c/dc^2 \ge 0$, i.e., $(d^2/dc^2)(dS/dt) \le 0$, are usually valid. Then, from (15) we find the optimum concentration c_0 from the known relations $\mu(c)$ and $P_c(c)$. It should be noted that μ and P_c depend, in turn, on the water saturation of the porous medium. Therefore, the efficiency of using solutions of surfactants can depend on the instantaneous water supply. This conclusion is confirmed by experimental results of [9]. A more rigorous analysis requires inclusion of adsorption, diffusion, and other transfer processes.

4. Thermal Methods. A similar procedure can be used for analysis of the thermal action on the bed. Here, we restrict ourselves to brief qualitative reasoning. In the general case, a term is added to the right-hand side of (5) that is proportional to $|\nabla T|^2$. Here dS/dt increases. However, as the temperature increases, the parameter μ decreases, which results in a decrease in dS/dt. Consequently, under these conditions, optimum heating is also possible.

5. Effect of Natural Magnetization of the Porous Medium. In accordance with the laws of magnetic hydrodynamics, in a flow of a conducting liquid (water) in a magnetic field various effects connected with the interaction of the electric and magnetic fields appear. In filtration of liquids the natural magnetic and electric fields are weak. However, in view of the small size of the pore channels and the large specific surface of the porous medium, it should be expected that magnetohydrodynamic effects can substantially affect the characteristics of filtration flows. This is indicated by the experimental results reported below.

Consider filtration of a homogeneous conducting liquid in the presence of an external magnetic field of strength $\overline{H}$. It is known that rocks have natural paleomagnetization. Therefore, in filtration of conducting liquids through real porous media electromagnetic phenomena that affect the characteristics of the flows appear. In the case of a liquid flow in an external magnetic field, the entropy production rate increases due the Joule effect. In the absence of an external electric field the contribution to the local rate of entropy production due to the Joule effect is $\sigma_l = c^{-2}R^{-1}T^{-1}(\overline{U} \times \overline{H})^2$ [5]. Substituting this expression into relation (5), we find

$$T\frac{dS}{dt} = -\int_{V_1} m\beta_1 P\frac{\partial P}{\partial t}\,dv - \int_{V_2} m\beta_2 P\frac{\partial P}{\partial t}\,dv + \frac{k^2}{\mu^2 c^2 R}\int_{V_1} (\nabla P \times \overline{H})^2\,dv + q_1 p_1 - q_2 p_2 .$$

It can be seen from the above relation that depending on the mutual orientation of the vectors $\overline{U}$ and $\overline{H}$, the rate of local entropy production varies, reaching a maximum when these vectors are mutually perpendicular. Therefore, it should be expected that the direction of filtration of a liquid affects the apparent hydrodynamic resistance because of a change in the dissipative component caused by the Joule effect.

It is of interest to compare these conclusions with experimental results on filtration of water through natural core samples of some oil fields obtained by the All-Russia Petroleum Research Institute [10]. Special measurements showed that the core samples used were homogeneous and isotropic, and under these conditions the penetration factor is a constant scalar. Table 1 contains permeabilities determined from measurements using a linear law of filtration. In the experiments mineralized water with a content of NaCl of 30 g/liter was used. A special procedure was used to determine the orientation of the main core sample relative to the direction of the meridian under natural conditions. Then, samples were cut out of this core in different directions and were subsequently used in the experiments. Average gas permeabilities in the "north-south" and "west-east" directions were determined from results of measurements. To eliminate the effect of possible errors in calculating the average increase in the water permeability, the latter was based on gas permeability determined in the same direction. As can be seen from Table 1, the observed permeability in filtration in the "west-east" direction is substantially higher than that in the "north-south" direction.

NOTATION

T, absolute temperature; S, entropy; I, generalized flow; X, generalized force; $\overline{U}$, filtration velocity; P, pressure; μ, viscosity; K, permeability; q, liquid discharge; m, porosity; β, compressibility; $K_1(\sigma)$, $K_2(\sigma)$, relative phase permeabilities for water and oil, respectively; σ, water saturation; L, distance between galleries; $l(t)$, coordinate of the displacement front; P_c, capillary pressure in the porous medium; c, concentration of the surfactant in water; R, specific resistance of the liquid; $\overline{H}$, magnetic-field strength; Q, rate of water injection; dv, volume element; dg, surface element.

REFERENCES

1. J. C. Slattery, The Theory of Momentum, Energy, and Mass Transfer in Continua [Russian translation], Moscow (1978).
2. V. N. Nikolaevskii, E. A. Bondarev, M. I. Mirkin, and G. S. Stepanova, Motion of Hydrocarbon Mixtures in Porous Media [in Russian], Moscow (1968).
3. C. de Groot and P. Mazur, Nonequilibrium Thermodynamics [Russian translation], Moscow (1964).
4. I. Prigogine, From the Existing to the Arising [Russian translation], Moscow (1985).
5. K. P. Gurov, Phenomenological Thermodynamics of Irreversible Processes [in Russian], Moscow (1978).
6. G. N. Barenblatt, V. M. Entov, and V. M. Ryzhik, The Theory of Unsteady Filtration of Liquids and Gases [in Russian], Moscow (1972).
7. M. L. Surguchev, Secondary and Ternary Methods of Increasing the Oil Recovery of Beds [in Russian], Moscow (1985).
8. R. Collins, The Flow of Liquids through Porous Materials [Russian translation], Moscow (1964).
9. I. M. Ametov, V. E. Gal'tsev, and A. M. Kuznetsov, Neftyanoe Khozyaistvo, No. 7, 43-44 (1995).
10. I. M. Ametov, A. G. Kovalev, A. M. Kuznetsov, V. V. Kuznetsov, and D. I. Sal'nikov, Neftyanoe Khozyaistvo, No. 6, 22-23 (1997).